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# QUANTITATIVE THEORY OF THE ELECTRIFICATION OF FALLING AEROSOL PARTICLES IN A ONE-DIMENSIONAL RISING AIR CURRENT

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Electrical phenomena in the lower strata of the atmosphere are known [1-3] to be determined essentially by the presence of ascending and descending air currents. These flows transport ions that exist in the atmosphere, where they are created mainly by radioactive emission. The atmospheric electric field, which also affects the motion of ions, in turn, depends itself on the concentration of those ions. The distribution of the ion concentrations and the electric field in rising air currents must be known, e.g., in calculating the charge of raindrops as a result of ion capture. The converse influence of drops on the ion and electric field distributions can be neglected in this case if the concentration of the drops is sufficiently small. Analogous phenomena are also encountered in the charging of aerosols in electrohydrodynamic devices that utilize special radioactive emission sources for the ionization of a gas [4, 5].

In this article we develop a theory to describe the distribution of the ion concentrations and electric field strength in one-dimensional air flows, as well as the electrification of falling aerosol particles in those flows in the case of a low particle concentration. The condition of one-dimensionality of the flow and the electric field is only approximately satisfied in a certain restricted zone of the air flow in practice. However, this customary assumption [1] makes it possible to formulate characteristic model problems that mirror extremely complicated natural and industrial processes. Their solution can be used as a basis for obtaining estimates of various quantities and pursuing qualitative studies of physical phenomena.

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1. Suppose that a gas containing ions with charges $\pm e$ ($e > 0$) move translationally with a specified constant velocity u, the direction of which coincides with the direction of the electric field vector E. We consider steady motions of the ions in the half-space $z \geq 0$, when all the flow parameters depend only on one coordinate z (the z axis is parallel to the vectors u and E). Such flows are described by the system of equations

$$\frac{dI_\pm}{dz} = \beta - \alpha n_+ n_-, \quad \frac{dE}{dz} = 4\pi e(n_+ - n_-), \tag{1.1}$$
$$I_\pm = -D_\pm \frac{dn_\pm}{dz} + n_\pm (u + b_\pm E).$$

Here $n_\pm$ denotes the concentrations of positive and negative ions; $b_\pm$ denotes their mobilities, which are related to the diffusion coefficients $D_\pm$ by the Einstein relation $b_\pm = \pm eD_\pm/(kT)$; $\alpha$ is the recombination coefficient, which is expressed in terms of the ion mobilities in dense gases according to the Langevin equation $\alpha = 4\pi e(b_+ - b_-)$; $\beta$ is the local rate of ion production as a result of, e.g., radioactive emission; and $n_0 = \sqrt{\beta/\alpha}$ is the equilibrium concentration of negative and position ions.

It follows from Eqs. (1.1) that the resultant (net) ion current $J = e(I_+ - I_-) = \text{const}$. We analyze flows for which $J = 0$. We specify the boundary conditions at $z = 0$ and $z = \infty$

$$I_\pm(0) = I_0, \; E(0) = E_0, \; n_\pm(\infty) = n_0. \tag{1.2}$$

From relations (1.1) and (1.2) we obtain $E(\infty) = 0$.

Next, we assume that $E_0$ is positive. This assumption does not detract from the generality of the problem statement. Indeed, the following substitution is allowed in the case $E_0 < 0$:

$$n_+^{(1)} = n_-, \; n_-^{(1)} = n_+, \; E^{(1)} = -E, \tag{1.3}$$
$$b_+^{(1)} = |b_-|, \; b_-^{(1)} = -b_+, \; E^{(1)}(0) = -E_0 > 0.$$

Once the problem (1.1), (1.2) has been solved for the quantities $n_\pm^{(1)}$ and $E^{(1)}$, the values of $n_\pm(z)$ and $E(z)$ are determined from Eqs. (1.3) by the inverse transformations.

We introduce the dimensionless variables

$$n_\pm^* = \frac{n_\pm}{n_0}, \; I_\pm^* = \frac{I_\pm}{n_0 u}, \; E^* = \frac{e\varkappa E}{kT}, \; z^* = \frac{z}{\varkappa}, \tag{1.4}$$

where $\kappa = \sqrt{kT/(8\pi e^2 n_0)}$ is the Debye radius. We use the dimensionless quantities (1.4) below, dropping the asterisks for convenience.

We first consider the case in which the ion mobilities are equal in absolute value. Then the following boundary-value problem is deduced from relations (1.1), (1.2), (1.4) for the dimensionless values of the charge $q = n_+ - n_-$ and the total ion concentration $n = n_+ + n_-$:

$$-\frac{d^2n}{dz^2} + \frac{d}{dz}(qE) + \mathrm{Pe}\frac{dn}{dz} = 2 + \frac{1}{2}(q^2 - n^2), \tag{1.5}$$
$$-\frac{d^2q}{dz^2} + \frac{d}{dz}(nE) + \mathrm{Pe}\frac{dq}{dz} = 0, \; \frac{dE}{dz} = \frac{q}{2};$$

$$n(0) + \mathrm{Re}_E^{-1} q(0) - \mathrm{Pe}^{-1}\frac{dn(0)}{dz} = 2I_0, \tag{1.6}$$
$$q(0) + \mathrm{Re}_E^{-1} n(0) - \mathrm{Pe}^{-1}\frac{dq}{dz} = 0,$$
$$E(0) = \mathrm{Pe}/\mathrm{Re}_E, \; n(\infty) = 2, \; q(\infty) = 0.$$

Here $Pe = u\kappa/D$ is the Péclet number with respect to the Debye radius and $Re_E = u/(|b_-|E_0)$ is the electrical Reynolds number.

In the case $E(0) = Pe/Re_E = 0$, $I_0 = 1$ the solution of the problem (1.5), (1.6) is trivial: $E \equiv 0$, $q \equiv 0$, $n \equiv 2$. Let us now suppose that a small deviation from this state takes place and $E(0) = \mathrm{Pe}/\mathrm{Re}_E \ll 1$, $|I_0 - 1| \ll 1$. The solution of the equations obtained for the perturbations of the unknown quantities $n' = n - 2$, $q'$, $E'$ by the linearization of relations (1.5) and (1.6) can be written in the form

$$q' = -\frac{2\,\mathrm{Pe}^2}{\mathrm{Re}_E} e^{-\mu z}, \quad n' = 4(I_0 - 1)\,\mathrm{Pe}^2 e^{-\lambda z},$$
$$E' = \frac{\mathrm{Pe}}{\mathrm{Re}_E} e^{-\mu z}, \quad \lambda \equiv -\frac{\mathrm{Pe}}{2} + \sqrt{\frac{\mathrm{Pe}^2}{4} + 2}, \quad \mu = -\frac{\mathrm{Pe}}{2} + \sqrt{\frac{\mathrm{Pe}^2}{4} + 1}. \tag{1.7}$$

The validity of the following asymptotic expressions is obvious:

$$\lambda = \sqrt{2} + o(\mathrm{Pe}), \quad \mu = 1 + o(\mathrm{Pe}), \quad \mathrm{Pe} \to 0,$$
$$\lambda = \frac{2}{\mathrm{Pe}} + o\left(\frac{1}{\mathrm{Pe}}\right), \quad \mu = \frac{1}{\mathrm{Pe}} + o\left(\frac{1}{\mathrm{Pe}}\right), \quad \mathrm{Pe} \to \infty. \tag{1.8}$$

It follows from the definition of the dimensionless coordinate $z^* = z/\kappa$, the solution (1.7), and relations (1.8) that the characteristic length $L_E$ of decay of the electric field is determined by the relations $L_E = \kappa$ for $Pe \ll 1$ and $L_E = Pe\,\kappa \gg \kappa$ for $Pe \gg 1$. In the latter case $L_E = Pe\,\kappa = u/(4\pi\sigma)$, where $\sigma = 2en_0b$ is the conductivity of the gas. This result can also be obtained directly from Eqs. (1.1) by discarding terms with the coefficients $D_\pm$. For ascending flows of pure atmospheric air under standard conditions, $n_0 \cong 5\cdot 10^8$ m$^{-3}$, $D_+ = 2.8\cdot 10^{-6}$ m$^2$/sec, $D_- = 4.3\cdot 10^{-6}$ m$^2$/sec, $e = 1.6\cdot 10^{-19}$ C, $u \cong 1$ m/sec, and so $\kappa \cong 4\cdot 10^{-2}$ m, $Pe \cong 10^4$, and $L_E \cong 4\cdot 10^2$ m.

A very high ion concentration ($n_0 \cong 5\cdot 10^{12}$ m$^{-3}$ [5]) is attained in electrohydrodynamic devices, owing to the use of special radioactive emission sources for the ionization of air. As a result, for the same values of $D_\pm$, u, e we obtain $\kappa \cong 4\cdot 10^{-4}$ m, $Pe \cong 10^2$, $L_E = Pe\,\kappa \cong 4\cdot 10^{-2}$ m. Consequently, $L_E \gg \kappa$ in both cases.

2. It is convenient to introduce new dimensionless variables for the investigation of flows with large values of the Péclet number:

$$z^{**} = z^*/\mathrm{Pe} = z/L_E, \quad E^{**} = E^*/\mathrm{Pe} = |b_-|E/u,$$
$$n_\pm^{**} = n_\pm^* = n_\pm/n_0, \quad I^{**} = I^* = I/(n_0 u). \tag{2.1}$$

We use the dimensionless quantities (2.1) below, dropping the asterisks for convenience.

Equations (1.1) and the boundary conditions (1.2), written in the new dimensionless variables (2.1), have the form ($\chi = b_+/|b_-|$)

$$\frac{dI_\pm}{dz} = \frac{1}{2}(1 + \chi)(1 - n_+ n_-),$$
$$I_+ = n_+(1 + \chi E) - \frac{\chi}{\mathrm{Pe}^2}\frac{dn_+}{dz}, \quad I_- = n_-(1 - E) - \frac{1}{\mathrm{Pe}^2}\frac{dn_-}{dz},$$
$$\frac{dE}{dz} = \frac{1}{2}(n_+ - n_-),$$
$$I_\pm(0) = I_0, \quad E(0) = E_0 = 1/\mathrm{Re}_E, \quad n_\pm(\infty) = 1. \tag{2.2}$$

Terms proportional to $1/Pe^2$ can be neglected in the limit $Pe \to \infty$. Then, invoking the integral

$$I_+ - I_- = n_+(1 + \chi E) - n_-(1 - E) = 0,$$

we arrive from Eqs. (2.2) at a boundary-value problem for determining the dimensionless values of the electrical charge density $q = n_+ - n_-$ and the electric field E as functions of the coordinate z:

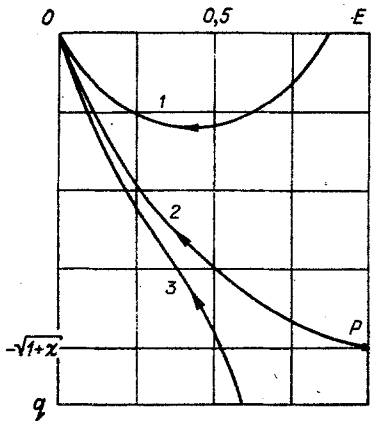


Fig. 1

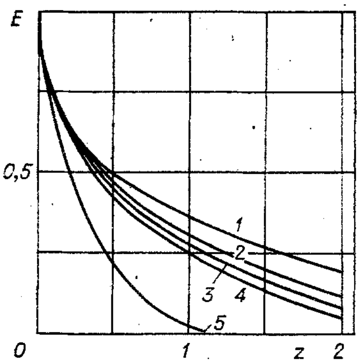


Fig. 2

$$\frac{dq}{dz} = \frac{[2+(\chi-1)E]q^2-(1+\chi)^2E}{2(1-E)(1+\chi E)E}, \quad \frac{dE}{dz} = \frac{1}{2}q, \; E(0) = E_0, \quad (2.3)$$
$$q(0) = (1+\chi)E_0I_0/[(E_0-1)(1+\chi E_0)].$$

Solving the problem (2.3), we find the concentrations of positive and negative ions according to the equations

$$n_+ = \frac{-(1-E)q}{(1+\chi)E}, \quad n_- = \frac{-(1+\chi E)q}{(1+\chi)E}, \quad (2.4)$$

which follows from the definition of $q = n_+ - n_-$ and the integral $I_+ - I_- = 0$.

It follows from Eqs. (2.4) that the solution of the problem (2.3) has physical significance only for $E_0 \leq 1$ (otherwise the value of $n_+$ becomes negative).

From (2.3) we obtain an equation for the function q(E):

$$\frac{dq}{dE} = \frac{[2+(\chi-1)E]q^2-(1+\chi)^2E^2}{(1-E)(1+\chi E)qE}. \quad (2.5)$$

A typical pattern of the integral curves of Eq. (2.5) is shown in Fig. 1 (the arrows indicate the direction in which the coordinate z increases). The integral curves corresponding to the initial value $I_0 = 0$ begin on the interval (0, 1) (curve 1) of the E axis. They are separated from the integral curves of the type 3 with an initial value $I_0 \neq 0$ by the separatrix 2, which emanates from the singular point P (of the "saddle-point" type) with coordinates $E = E(0) = 1$, $q = q(0) = -\sqrt{1+\chi}$ ($I_0 = 0$ on the separatrix). All the integral curves terminate at the zero point $E = 0$, $q = 0$, which corresponds to the limit $z \to \infty$. It is seen that the field E always decreases with increasing z.

If the mobilities of positive and negative ions are equal in absolute value ($|b_-| = b_+$, $\chi = 1$), Eq. (2.5) is analytically integrable. In this case the solution of the problem (2.3) has the form

$$q = \frac{-2E}{1-E}\left[1+(I_0^2-1)\frac{E^2}{E_0^2}+2E^2\ln\frac{E}{E_0}\right]^{\frac{1}{2}},$$
$$z = \int_{E_0}^{E}\frac{E^2-1}{E}\left[1+(I_0^2-1)\frac{E^2}{E_0^2}+2E^2\ln\frac{E}{E_0}\right]^{-\frac{1}{2}}dE.$$

If $\chi \neq 1$, the problem (2.3) must be solved numerically. The results of such a computation in the form of curves E(z) for values of $\chi$ = 0.1, 0.71, 1, 1.4, and 10 (curves 1-5, respectively) are shown in Fig. 2. It is assumed that $E_0 = 1$ and $I_0 = 1$. It is seen that large values of the parameter $\chi$ correspond to a more rapid decay of E with increasing z. For atmospheric dry air under standard conditions it can be assumed that $b_+ = 1.37\cdot10^{-4}$ $m^2/(sec\cdot V)$, $b_- = -1.9\cdot10^{-4}$ $m^2/(sec\cdot V)$, and $\chi = 1.37/1.0 \cong 0.71$ [3]. If $E_0 < 0$, we can transform (1.2) and, under the stated assumptions for the dimensionless quantities $E^{(1)} = -E/(b_+u)$ and $q^{(1)} = (n_- - n_+)/n_0$, arrive at the boundary-value problem (2.3) with the value $\chi = 1.9/1.37 \cong 1.4$.

3. Let us assume that an initially uncharged spherical conducting particle (drop) of radius a falls under the action of gravity in the investigated flow with a constant velocity $v > 0$ in the direction opposite to the flow. The dependence of the particle charge $e_p$ on the coordinate z is described by the equation

$$-v\frac{de_p}{dz} = J_+ + J_-, \quad e_p(\infty) = 0, \tag{3.1}$$

in which $J_\pm$ denotes the ion currents impinging on the particle.

The electric field and ion concentration in the flow are determined from the solution of the problem (2.3). According to the results of Sec. 2, the condition $E|b_-|/u < 1$ holds everywhere in the flow. Accordingly, the velocity u + v of the gas flow around the particle is always greater than $|b_-|E$. The following expressions hold in this case for the positive and negative ion currents $J_\pm$ impinging on the particle as a result of ion diffusion [6, 7]:

$$J_+ = \begin{cases} \dfrac{\pi e b_+ n_+}{3a^2E}(e_p - 3a^2E)^2, & |e_p| \leqslant 3a^2E, \\ -4\pi e b_+ n_+ e_p, & e_p < -3a^2E, \\ 0, & e_p > 3a^2E, \end{cases} \tag{3.2}$$

$$J_- = \begin{cases} -4\pi e|b_-|n_- e_p, & e_p \geqslant 0, \\ 0, & e_p < 0. \end{cases}$$

The difference in the expressions for $J_\pm$ is attributable to the following. Although both types of ions impinge on the particle from below, they settle on different parts of its surface. For example, in the case $3a^2E > e_p \geq 0$ positive ions settle only on the underside of the particle, while negative ions move around the particle and settle primarily on its topside [3, 6, 7], because for them $b_-(E\nu) > 0$ at the lower critical point ($\nu$ is the outward normal to the surface of the particle). For $e_p < 0$, positive ions settle partially on the topwise of a drop, while negative ions move around it and are carried away by the air flow without settling [3, 6, 7]. Equations (3.2) are derived both for Stokes flow (Re << 1) and for nonseparating potential flow (Re >> 1) of a spherical particle [3, 6, 7] and are customarily used for investigations of the electrification of aerosol particles over a wide range of Re [1-3, 6].

It is evident from Eqs. (3.2) that an initially uncharged particle in the investigated flow can acquire only a positive charge satisfying the condition $0 < e_p < 3a^2E$.

Below, we use the dimensionless variables

$$e_p^{**} = \frac{|b_-|e_p}{3a^2E}, \quad v^{**} = \frac{v}{u}. \tag{3.3}$$

We drop the asterisk for convenience.

Equation (3.1) in the dimensionless variables (2.1), (3.3) has the following form with allowance for the current expressions (3.2) and the inequality $0 < e_p < 3a^2E$:

$$v\frac{de_p}{dz} = -\frac{1}{8}\chi n_+ E\left(1 - \frac{e_p}{E}\right)^2 + \frac{1}{2}n_- e_p, \quad e_p(\infty) = 0, \tag{3.4}$$

where $n_\pm$ and E are specified functions of the coordinate z, which are determined according to Eqs. (2.4) on the basis of the solution of the problem (2.3) with fixed initial values of $I_0$ and $E_0$. From the integral $I_+ - I_- = 0$ we obtain the relation

$$n_+ = \frac{1 + \chi E}{1 - E}n_-.$$

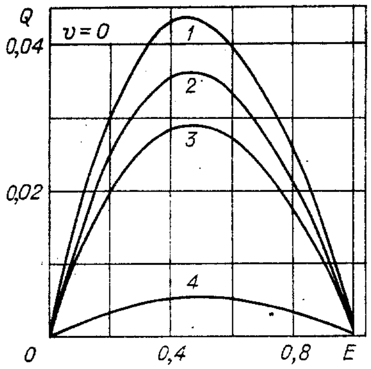

Fig. 3

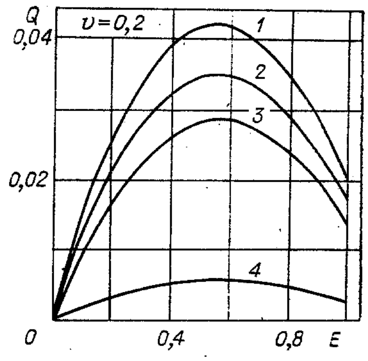

Fig. 4

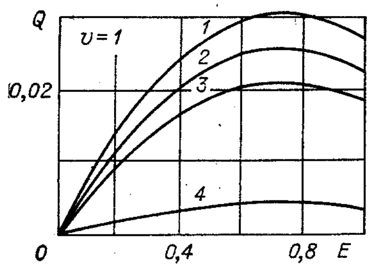

Fig. 5

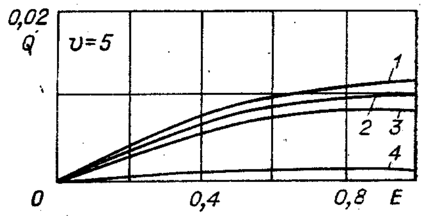

Fig. 6

We use it to deduce the following from Eq. (3.4) and the second equation (2.3):

$$\frac{de_p}{dE}=\frac{1}{4(1+\chi)v}\left[\chi(1-E)\left(1-\frac{e_p}{E}\right)^2-4(1+\chi E)\frac{e_p}{E}\right],\quad e_p|_{E=0}=0.$$

The solution of Eq. (3.5) yields a universal function $e_p(E, v, \chi)$, which is the same for all solutions of the problem (2.3) with different initial values of $I_0$ and $E_0$.

The results of the numerical integration of Eq. (3.5) for various values of the constants $\chi$ and v are shown in Figs. 3-6, where the point E = 0, $e_p$ = 0 corresponds to the limit $z \to \infty$ and curves 1-4 corresponds to $\chi$ = 1.4, 1, 0.71, 0.1.

The function $e_p(E)$ is nonmonotonic, and it has a maximum corresponding to the maximum charge acquired by the particle in its motion. For a mixed value of the parameter $\chi = b_+/|b_-|$ the maximum charge decreases as the dimensionless velocity v is increased (the particle "has less time" to become charged) and is the greatest for equilibrium charge, $v \to 0$ (Fig. 3). For a fixed value of the parameter v the dimensionless charge $e_p$ for a given value of E decreases as the ratio $b_+/|b_-| = \chi$ is increased. All the foregoing considerations are based on the assumption $1 \geq E_0 > 0$. If $-1 \leq E_0 < 0$, the particle becomes negatively charged. In this case, writing the corresponding expressions for $J_+$ in (3.1) and then making the change of variables (2.1), we arrive at Eq. (3.5) with the value of the constant $\chi = |b_-|/b_+$, which determines the dimensionless charge $e_p^{(1)} = -b_+e_p/(3a^2u)$ as a function of the dimensionless electric field $E^{(1)} = -b_+E/u$. For example, the curves corresponding to $\chi$ = 1.4 in Figs. 3-6 characterize both the function $e_p^{**}(E^{**})$ for $E_0 > 0$, $b_+/|b_-| = 1.4$ and the function $e_p^{(1)}(E^{(1)})$ for $E_0 < 0$, $b_+/|b_-| = 1/1.4 = 0.714$.

As an example, we consider an aerosol particle of radius $a = 1.5\cdot10^{-4}$ m falling in an ascending air flow ($b_- = -1.9\cdot10^{-4}$ m²/(sec·V), $\chi$ = 0.71, u = 1.2 m/sec). The maximum charge on the particle during slow descent (v << u) is equal to $2\cdot10^{-15}$ C. The height $L_m$ at which the particle acquires its maximum charge also depends on the quantities $E_0$, $n_0$, $I_0$, e. In particular, for $E_0 \cong 6\cdot10^3$ V/m, $n_0 \cong 5\cdot10^8$ m$^{-3}$, $I_0 = n_0u = 6\cdot10^8$ m$^{-2}$ sec$^{-1}$, $e = 1.6\cdot10^{-19}$ C we obtain $L_m \cong 2\cdot10^2$ m; for $n_0 \cong 5\cdot10^{12}$ m$^{-3}$, $I_0 = 6\cdot10^{12}$ m$^{-2}$ sec$^{-1}$, and the same values of $E_0$, e as before, we have $L_m \cong 2\cdot10^{-2}$ m. In further descent, the particle loses charge due to the capture of positive ions.

In the above-considered one-dimensional flows the parameters α, β, $b_{\pm}$, $D_{\pm}$, u and v have been regarded as constant. This is justified in application to atmospheric phenomena for $L_E \lesssim 10^2$ m. For significantly larger values of $L_E = \kappa Pe$, generally speaking, it is necessary to take into account the height dependence of the indicated parameters, along with the nonuniformity of the air currents.

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